Lecture 5:
Pricing Factors

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Notation

notation	description
$\tilde{r}$	excess return rate over the period
$\tilde{r}^i$	arbitrary asset i
$\tilde{r}^p$	arbitrary portfolio p
$\tilde{r}^{\mathrm{t}}$	tangency portfolio
$\tilde{r}^m$	market portfolio
$\tilde{r}^s$	size portfolio
$\tilde{r}^v$	value portfolio
$\beta^{i,j}$	regression beta of $\tilde{r}^i$ on $\tilde{r}^j$

Outline

Fama-French Factors

Momentum

APT

Economic Factors (CCAPM)

Appendix: PCA

Fama-French model

The Fama-French 3-factor model is one of the most well-known multifactor models.

$$\mathbb{E}\left[\tilde{r}^i\right] = \beta^{i,m}\,\mathbb{E}\left[\tilde{r}^m\right] + \beta^{i,s}\,\mathbb{E}\left[\tilde{r}^s\right] + \beta^{i,v}\,\mathbb{E}\left[\tilde{r}^v\right]$$

- $\tilde{r}^m$ is the excess market return as in the CAPM.
- $\tilde{r}^s$ is a portfolio that goes long small stocks and shorts large stocks.
- $\tilde{r}^v$ is a portfolio that goes long value stocks and shorts growth stocks.

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Use of growth and value

The labels “growth” and “value” are widely used.

- Historically, value stocks have delivered higher average returns.
- So-called “value” investors try to take advantage of this by looking for stocks with low market price per fundamental or per cash-flow.
- Much research has been done to try to explain this difference of returns and whether it is reflective of risk.
- Many funds (ETF, mutual funds, hedge funds) orient themselves around being “value” or “growth”.

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FF Measure of Value

The **book-to-market** (B/M) ratio is the market value of equity divided by the book (balance sheet) value of equity.

- High B/M means strong (accounting) fundamentals per market-value-dollar.
- High B/M are value stocks.
- Low B/M are growth stocks.

For portfolio value factor, this is the most common measure.

Other value measures

Many other measures of value based on some cash-flow or accounting value per market price.

- Earnings-price is a popular metric beyond value portfolios. Like B/M, the E/P ratio is accounting value per market valuation.
- EBITDA-price is similar, but uses accounting measure of profit that ignores taxes, financing, and depreciation.
- Dividend-price uses common dividends, but less useful for individual firms as many have no dividends.

Many other measures, and many competing claims to special/better measure of 'value'.

Other Popular Factors

Sort portfolios of equities based on...

- Price movement. Momentum, mean reversion, etc.
- Volatility. Realized return volatility, market beta, etc.
- Profitability.*
- Investment.*

*As measured in financial statements.

Characteristics or Betas?

LFPM says security's beta matters, not its measure of the characteristic.

- So what does FF model expect of a stock with high B/M yet low correlation to other high B/M stocks?
- Beta earns premium—not the stock's characteristic.
- This is one difference between FF "value" investing and Buffett-Graham "value" investing.

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Testing the model

Testing these LFMs is analogous to testing the CAPM.

- Time-series test.
- Cross-sectional test.
- Statistical significance through chi-squared test of alphas. (ie Do the factors span the MV frontier?)

Finding the right factors

Hundreds of tests and papers written about LFMs!
Does z^j help the model given the other $\boldsymbol{z}$?

- Really asking whether z^j adds to the MV frontier generated by $\boldsymbol{z}$.
- Calculate factor MV:

$$\boldsymbol{w} = \boldsymbol{\Sigma}_{\boldsymbol{z}}^{-1} \boldsymbol{\lambda}_{\boldsymbol{z}} \frac{1}{\gamma}$$

- Any significant weight on factor z^j?
- Easy to formally test this using t-stat, chi-squared test, etc.

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Return autoregressions: momentum or reversion?

With the overall market index, there is no clear evidence of momentum or mean-reversion.

$$r^m_{t+1} = \alpha + \beta r^m_t + \epsilon_{t+1}$$

The autoregression[1] does not find β to be significant, (statistically, economically.)

[1] Of course, we can write this regression as

$$\left(r^m_{t+1} - \mu\right) = \beta \left(r^m_t - \mu\right) + \epsilon_{t+1}$$

where μ is the mean of r^m, and $\alpha = (1 - \beta)\mu$.

Autocorrelation of individual stocks

What about individual stocks? Is there significant autocorrelation in their returns?

- At a monthly level, most equities would have no higher than $\beta = 0.05$.
- Thus, for a long time the issue was ignored; too small to be economical—especially with trading costs!

Trading on small autocorrelation

Two keys to taking advantage of this small autocorrelation:

1. Trade the extreme "winners" and "losers"
   - Small autocorrelation multiplied by large returns gives sizeable return in the following period.
   - By additionally shorting the biggest "losers", we can magnify this further.
2. Hold a portfolio of many "winners" and "losers."
   - By holding a portfolio of such stocks, diversifies the idiosyncratic risk.
   - Very small R^2 stat for any individual autoregression, but can play the odds (ie. rely on the small R^2) across 1000 stocks all at the same time.

Illustration: Workings of momentum

- Assume each stock i has returns which evolve over time as

$$\left(r^i_{t+1} - \underbrace{0.83\%}_{mean}\right) = \underbrace{0.05}_{autocorr}\left(r^i_t - \underbrace{0.83\%}_{mean}\right) + \epsilon_{t+1}$$

- Assume there is a continuum of stocks, and their cross-section of returns for any point in time, t, is distributed as

$$r^i_t \sim \mathcal{N}\left(0.83\%, 11.5\%\right)$$

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Illustration: normality

From the normal distribution assumption,

- The top 10% of stocks in any given period are those with returns greater than 1.28σ.
- Thus, the mean return of these "winners" is found by calculating the conditional mean:

$$\mathbb{E}\left[r \mid r > 1.28\sigma\right] = \frac{\int_{1.2816}^{\infty} r\phi(r)dr}{\int_{1.2816}^{\infty} \phi(r)dr}$$

  where $\phi(x)$ is the pdf of the normal distribution listed above.

- For a normal distribution, we have a closed form solution for this conditional expectation, (mean of a truncated normal,)

$$\mathbb{E}\left[r \mid r > 1.28\sigma\right] = 1.755\sigma = 21.01\%.$$

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Illustration: autocorrelation

From the autocorrelation assumption:

- A portfolio of time t winners, r^u, is expected to have a time $t+1$ mean return of

$$\mathbb{E}_t\left[r^u_{t+1}\right] = 0.83\% + .05\left(1.755\sigma - 0.83\%\right) = 1.84\%$$

- We assumed that the average return across stocks is 0.84%.
- Thus, the momentum of the winners yields an additional 1% per month.
- Going long the winners as well as short the losers doubles this excess return.

Implementing a momentum strategy over time

A momentum strategy with equities is formed by ranking securities on recent realized return.

- Go long on the portfolio of recent periods's biggest winners and go short recent period's biggest losers.
- After holding the "momentum" portfolio for some time period, re-rank the "winners" and "losers".
- Re-sorting frequently is important as the securities move frequently in and out of "winner/loser" rankings.

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Updating the rankings

Dropped			Added		
Ticker	Sep14	Oct14	Ticker	Sep14	Oct14
AAPL	47.93%	32.97%	ADSK	33.84%	45.50%
CMG	55.45%	28.22%	ALNY	22.01%	37.08%
DECK	47.42%	31.69%	CDNS	27.39%	37.98%
FSLR	63.67%	21.61%	CDW	36.01%	39.05%
JLL	44.72%	31.54%	CFN	22.63%	46.54%

- 5 of the 17 stocks which moved in and out of “winners” of the Russell 1000. (ie. Joined or dropped from top-10% of the index.)
- Ranked by cumulative one-year return from Oct. 2013 - Sep. 2014, and then re-ranked one month later based on cumulative return from Nov. 2013 - Oct 2014.

Trading costs versus momentum returns

Resorting frequency must balance two objectives:

- Minimizing trading costs.
- Updating portfolio to hold highest-momentum assets.

For US Equities, monthly excess returns up to 0.67% per month—before trading costs.

Trading costs

Often claimed that momentum does not survive net of trading costs.

- Transaction costs.
  - Transaction costs would be overwhelming for a retail investor.
  - But institutional investors have much smaller costs.
  - Can delay or adjust portfolio rebalancing to lessen turnover.
- Tax burden.
  - Lots of trading may induce large capital gains taxes.
  - But selling losers, (reaping capital losses) and holding winners (delaying capital gains.)
  - Also, momentum stocks tend to have relatively low dividend yields, avoiding inefficient dividend taxation.

Widespread momentum

Momentum strategies in many asset classes deliver excess returns.

- International equities and equity indices
- Government bonds
- Currencies
- Commodities
- Futures

Evidence: Momentum returns

Table: Excess returns to momentum strategies

	Excess return	CAPM alpha	Sharpe ratio
U.S. stocks	5.8%	7.2%	0.86
Global stocks	5.3%	5.8%	1.21
Currencies	5.6%	5.7%	0.69
Commodities	17.1%	17.1%	0.77

- Source: Asness, et.al. 2013. Table 1.
- Annualized estimates. Monthly data, 1972-2011.
- See paper for t-stats.

Risk-based explanations

Is momentum strategy associated with some risk?

- Volatility?
- Correlation to market index, such as the S&P?
- Business-cycle correlation?
- Tail risk?
- Portfolio rebalancing risk?

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Behavioral explanations

Can investor behavior explain momentum?

- Under-reaction to news.
  - At time t, positive news about stock pushes price up 5%.
  - At time $t+1$, investors fully absorb the news and stock goes up another 1% to rational equilibrium price.
- Over-reaction to news.
  - At time t, positive news about stock pushes price up 5%—to rational equilibrium.
  - At time $t+1$, investors are overly optimistic about the news and recent return. Stock goes up another 1%.

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Explaining momentum

Years of debate regarding the explanation for momentum.

- Any evidence for the rational explanation? Can we specify the risk that makes investors reluctant to engage in momentum strategies?
- Suppose we believe the cause is behavioral. How can we distinguish between the two, (opposite!) behavioral theories on the previous slide?

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Momentum in practice

Momentum is one of the most popular strategies used by managed funds.

- The lack of a perfect explanation of momentum has not kept funds from using it!
- It is popular not just for the large excess returns but also due to its potential help in diversification—given its low correlation with other popular strategies, (such as value-investing.)
- Even accessible to retail investors through mutual-fund-type products.

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The APT

Arbitrage pricing theory (APT) gives conditions for when a Linear Factor Decomposition of return **variation** implies a Linear Factor Pricing for **risk premia**.

- The assumptions needed will not hold exactly.
- Still, it is commonly used as a way to build LFP for risk premia in industry.

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APT factor structure

Suppose we have some excess-return factors, $\mathbf{x}$, which work well as a LFD[2].

$$\tilde{r}^i_t = \alpha^i + \left(\boldsymbol{\beta}^{i,\mathbf{x}}\right)' \mathbf{x}_t + \epsilon^i_t$$

APT Assumption: The residuals are uncorrelated across regressions

$$\text{corr}\left[\epsilon^i, \epsilon^j\right] = 0, \quad i \neq j$$

That is, the factors completely describe return comovement.

[2] We continue using the factor notation, x_t, though we could more explicitly write $\tilde{r}_t$, given that the factors are themselves excess returns.

A Diversified Portfolio

Take an equally weighted portfolio of the n returns

$$\begin{aligned}\tilde{r}_t^p =& \frac{1}{n}\sum_{i=1}^{n}\tilde{r}_t^i \\ =& \alpha^p + \left(\beta^{p,\mathbf{x}}\right)' \mathbf{x}_t + \epsilon_t^p\end{aligned}$$

where

$$\alpha^p = \frac{1}{n}\sum_{i=1}^{n}\alpha^i, \qquad \beta^{p,\mathbf{x}} = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{\beta}^{i,\mathbf{x}}, \qquad \epsilon^p = \frac{1}{n}\sum_{i=1}^{n}\epsilon_t^i$$

Idiosyncratic variance

The idiosyncratic risk of $\tilde{r}_t^p$ depends only on the residual variances.

- By construction, the residuals are uncorrelated with the factors, $\mathbf{x}$.
- By assumption, the residuals are uncorrelated with each other.

$$\text{var}\left[\epsilon^p\right] = \frac{1}{n}\overline{\sigma_\epsilon}^2$$

where $\overline{\sigma_\epsilon}^2$ is the average variance of the n assets.

Perfect factor structure

As the number of diversifying assets, n, grows

$$\lim_{n\to\infty} \text{var}\left[\epsilon^{p}\right] = 0$$

Thus, in the limit, $\tilde{r}^{p}$ has a perfect factor structure, with no idiosyncratic risk:

$$\tilde{r}^{p}_{t} = \alpha^{p} + \left(\beta^{p,\mathbf{x}}\right)' \mathbf{x}_{t}$$

This says that $\tilde{r}^{p}$ can be perfectly replicated with the factors $\mathbf{x}$.

Obtaining the LFP in $\mathbf{x}$

APT Assumption 2: There is no arbitrage.

Given that $\tilde{r}^p$ is perfectly replicated by the return factors, $\mathbf{x}$, then

$$\alpha^p = 0$$

Thus, taking expectations of both sides, we have a LFP:

$$\mathbb{E}\left[\tilde{r}^p\right] = \left(\beta^{p,\mathbf{x}}\right)' \boldsymbol{\lambda}^x$$

where

$$\boldsymbol{\lambda}^x = \mathbb{E}\left[\mathbf{x}\right]$$

Explaining variation and pricing

The APT comes to a stark conclusion:

- Assume we find a Linear Factor Decomposition (LFD) that works so well it leaves no correlation in the residuals.
- That is, the set of factors explains realized returns across time. (Covariation)
- The APT concludes the factors must also describe expected returns across assets. (Risk premia)

That is, a perfect LFD will also be a perfect LFP!

Outline

Non-return factors

What if we want to use a vector of factors, $\boldsymbol{z}$, which are not themselves assets?

- Examples include slope of the term structure of interest rates, liquidity measures, economic indicators, etc.

- The time-series tests of LFM relied on,

$$\lambda_{\boldsymbol{z}} = \mathbb{E}\left[\tilde{\boldsymbol{r}}^{z}\right], \qquad \boldsymbol{\alpha} = \boldsymbol{0}$$

  But with untraded factors, $\boldsymbol{z}$, we do not have either implication.

- Thus to test an LFM with untraded factors, we must do the cross-sectional test.

The CCAPM

The Consumption CAPM (CCAPM) says that the only systematic risk is consumption growth.

$$\mathbb{E}\left[\tilde{r}^i\right] = \beta^{i,c}\,\lambda_c$$

where c is some measure of consumption growth.

- The challenge is specifying a good measure for c.
- The CAPM can be seen as a special case where $c = \tilde{r}^m$.
- Generally, measures of c is a non-traded factor.
- We could build a replicating portfolio, or test it directly in the cross-section.

Testing the CCAPM across assets

1. Run the time-series regression for each test-security, i.

$$\tilde{r}^i_t = a^i + \beta^{i,c} c_t + \epsilon^i_t$$

   The intercept is denoted a to emphasize it is not an estimate of model error, α.

2. Run the single cross-sectional regression to estimate the premium, λ_c and the residual pricing errors, α^i.

$$\mathbb{E}\left[\tilde{r}^i\right] = \lambda_c \, \beta^{i,c} + \alpha^i$$

   As usual, the theory implies the cross-sectional regression should not have an intercept, but it is often included.

Evidence for CCAPM: consumption beta and returns

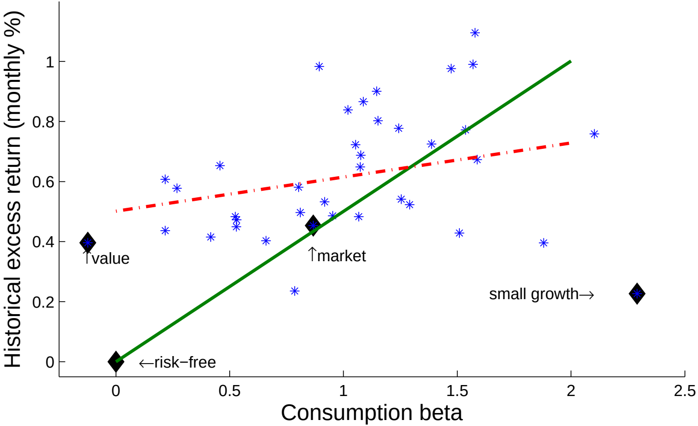

Figure: Data Source: Ken French, Federal Reserve. Monthly, 1959-2010.

Model with alternate consumption measurement

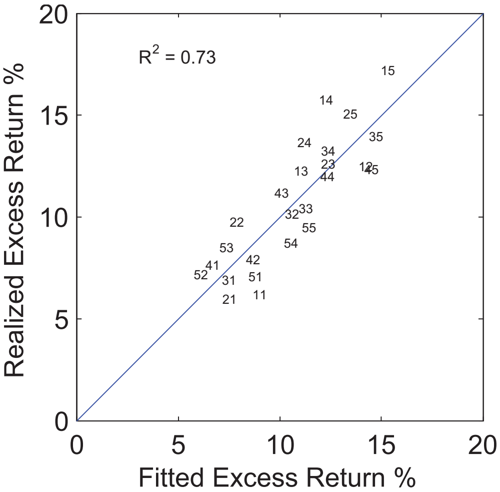

Figure: Jagannathan and Wang (2005).

Macro factors

A number of industry models use non-traded, macro factors.

- GDP growth
- Recession indicator
- Monetary policy indicators
- Market volatility

Consumption factors are widely studied in academia, but less in industry.

Factor-mimicking returns

Factor-mimicking returns are the linear projection of non-return factors onto the space of traded returns, $\boldsymbol{r}$:

$$\tilde{\boldsymbol{r}}^{z} = \mathbb{L}\left(\boldsymbol{z} \mid \boldsymbol{r}\right)$$

Recall that a linear projection can be calculated simply by regressing $\boldsymbol{z}$ on the available security returns, $\boldsymbol{r}$.

- If there is a LFM in $\boldsymbol{z}$, then there is also a LFM in the factor-mimicking portfolios, $\tilde{\boldsymbol{r}}^{z}$.
- Then we are back to having an LFM in tradable factors, $\tilde{\boldsymbol{r}}^{z}$.

Outline

Principal components

The **principal components** of returns are statistical factors which maximize the amount of return variation explained.

- $\tilde{\boldsymbol{r}}$ denotes an $n \times 1$ random vector of excess returns with covariance matrix $\boldsymbol{\Sigma}$.
- The first principal component of returns, x is characterized by a vector of excess return loadings, $x_t^1 = \mathbf{q}_1' \tilde{\boldsymbol{r}}_t$ which solves,

$$\max_{\mathbf{q}} \ \mathbf{q}'\boldsymbol{\Sigma}\mathbf{q}$$
$$\text{s.t. } \mathbf{q}'\mathbf{q} = 1$$

- Thus, $x_t^1 = \mathbf{q}_1' \tilde{\boldsymbol{r}}_t$ is the portfolio return with maximum variance.

General definition of principal components

The ith principal component, $x_t^i = \mathbf{q}_i' \tilde{\boldsymbol{r}}_t$, has loading vector, $\mathbf{q}_i$, solves the same problem as above, but with the additional constraint that it be uncorrelated to the previous $i-1$ principal components:

$$\max_{\mathbf{q}} \ \mathbf{q}' \boldsymbol{\Sigma} \mathbf{q}$$

$$\text{s.t. } \mathbf{q}' \mathbf{q}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

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Eigenvector decomposition

The covariance matrix of returns has the following eigenvector decomposition:

$$\mathbf{\Sigma} = \mathbf{Q}'\Psi\mathbf{Q}$$

- $\mathbf{Q}$ is an $n \times n$ matrix where each column is an eigenvector, q_i.
- Ψ is an $n \times n$ diagonal matrix of eigenvalues, ψ_i.
- The eigenvectors are orthonormal: $\mathbf{Q}'\mathbf{Q} = \mathcal{I}$.

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Eigenvectors as principal components

It turns out that the solution to the principal components problem is given by the eigenvectors of Σ.

- The variance of principal component i is

$$\text{var}\left[x^i\right] = \mathbf{q}_i'\mathbf{\Sigma}\mathbf{q}_i = \psi_i$$

- The first principal component has maximum variance, so its weight vector is the eigenvector associated with the largest eigenvalue.

Factor model of principal components

Not only do we have the principal component factors as linear combinations of the returns,

$$\mathbf{x}_t = \mathbf{Q}' \tilde{r}_t$$

But we can multiply both sides by $\mathbf{Q}$ to find that returns can be decomposed into a linear combination of the principal components:

$$\tilde{r}_t = \mathbf{q}_1 x_t^1 + \mathbf{q}_2 x_t^2 + \ldots + \mathbf{q}_n x_t^n$$

Of course, using n factors to describe returns on n assets is not useful.

Reduction in factors

- The point of principal component models is to use a much smaller subset of the principal components to explain most of the variation.
- For instance, one might use just three principal components in order to describe the variation of 20 or 50 different return series.

Selecting the PC model

Consider that the percent of the variance of returns explained by principal component i is

$$\frac{\psi_i}{\sum_{j=1}^{n} \psi_j}$$

Consider the percent of total variation explained by just these k PC factors:

$$\frac{\sum_{j=1}^{k} \psi_j}{\sum_{j=1}^{n} \psi_j}$$

If a subset of k can explain most of the variation, this may be a good factor decomposition for the return variation.

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