Case:
Barnstable and Long-Run Risk

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FINM 36700: Portfolio Theory

Barnstable strategy

- Want to maximize long-run return.
- Not worried about short-run fluctuations.
- No diversification: all-equity strategy.

Shortfall calculation

$$r_{t,t+h} \sim \mathcal{N}\left(h\mu, h\sigma^2\right), \qquad \bar{r}_h \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{h}\right)$$

Both imply, probability of stock under-performance:

$$\Phi\left(-\sqrt{h}\,\frac{\mu - r_f}{\sigma}\right)$$

where Φ denotes the cdf of the standard normal distribution.

Numbers here are old: Shortfall probabilities

Table: Probability of shortfall

	Barnstable	Full-sample
parameters		
μ	0.1094	
σ	0.1600	
years		
10	0.1644	
20	0.084	
30	0.0454	

Standard error on μ is 0.021.

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Probability of shortfall - Barnstable parameters

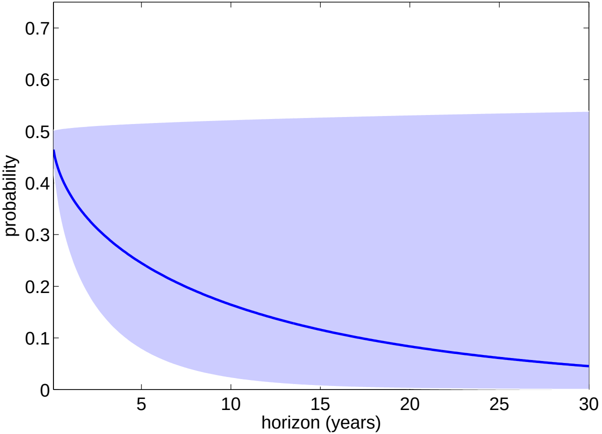

Figure: Probability that cumulative equity return underperforms assumed cumulative risk-free return of 6% per year. Based on Barnstable's parameter estimates. Shaded area shows shows calculation using $\hat{\mu}$ estimates from 95% confidence interval on μ.

Probability of shortfall - Full-sample parameters

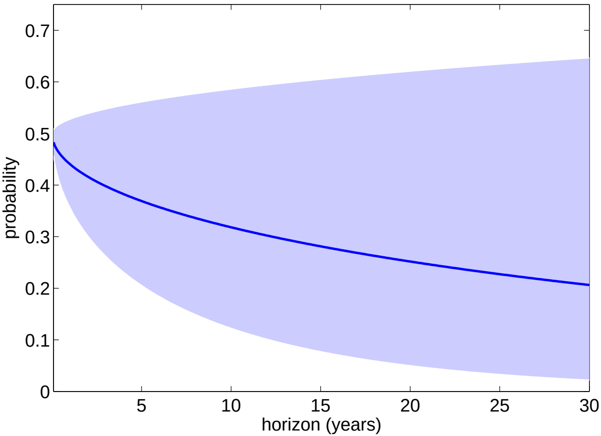

Figure: Probability that cumulative equity return underperforms cumulative risk-free rate. Based on sample estimates of parameters, with sample from Jan. 1928 to Dec. 1999.

Put probabilities - full-sample estimates

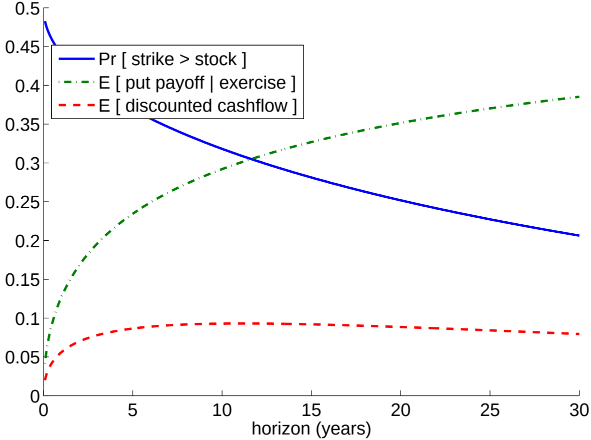

Figure: Expected exercise, conditional payoff, and cashflow to put. Using full-sample parameters.

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Put probabilities - supposing $\mu = 0.055$

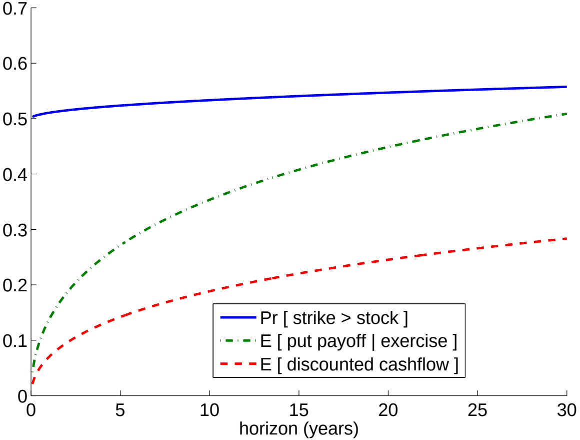

Figure: Expected exercise, conditional payoff, and cashflow to put. Using $\mu = .055$, which is within confidence interval of μ estimate.

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Not-Required Material: Valuation dependent on σ

The Black-Scholes formula for a put option on a stock, S.

- Volatility, σ, is constant
- The strike, X, grows at the constant riskless rate r_f.

$$\frac{P(\tau)}{S_0} = \Phi(-d_-) - \Phi(-d_+)$$

$$d_+ = \frac{1}{2}\sqrt{\tau}\sigma, \qquad d_- = -\frac{1}{2}\sqrt{\tau}\sigma$$

$$\Rightarrow \frac{P(\tau)}{S_0} = 2\Phi\left(\frac{1}{2}\sqrt{\tau}\sigma\right) - 1$$

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Puts versus trust

- With the trust, Barnstable has a payoff of,

$$\max\left(R_{30}^{\text{mkt}} - \exp\{.06(30)\},\ 0\right)$$

where R_{30}^{mkt} denotes the cumulative market return from 1999-2029.

- If Barnstable pursues the put strategy, the payoff at maturity of the put is

$$-\max\left(\exp\{.06(30)\} - R_{30}^{\text{mkt}}, 0\right)$$

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Preferred shares

- The preferred shares can be replicated by going long a riskless 6% and short a put option on R_{30}^{mkt}.
- So the payoff is

$$\exp\{(.06)30\} - \max\left(\exp\{.06(30)\} - R_{30}^{\text{mkt}}, 0\right)$$

- One can verify that the payoff of the preferred and common shares indeed sums to the payoff of R_{30}^{mkt}.

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Afterward: Probability of exercise

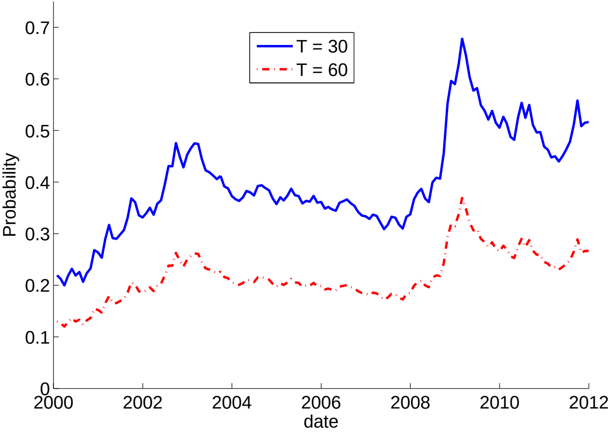

Figure: Probability given date t, that cumulative market return from 1999-2029 under-performs 30-year cumulative return of the 6% benchmark. $\Pr\left(R_{30}^{\text{mkt}} < \exp\{.06\,(30)\}\right)$.

Afterward: underperformance

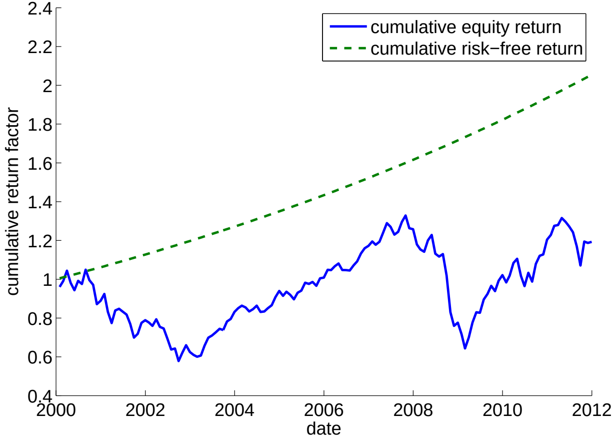

Figure: Relative performance of the equity market compared to the 6% risk-free investment.

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